Richmond's [8] formulation, where pulse basis functions were employed for the solution of the electric field integral equation.

In [7], Peterson and Klock presented a solution of an improved magnetic field integral equation by employing triangular elements with linear basis functions in the TE case. Here we compare our results with those in [7] for a dielectric cylinder with a radius 0.05λ having a relative permittivity $\epsilon_r = 4.0 - j100.0$. The results are shown in Fig. 5 and, as seen, the present approach provides a higher accuracy.

V. Conclusion

In this paper, we examined three integral equations for TE scattering by dielectric cylinders having large values of permittivity. A moment method solution of the volume–surface integral equation was then developed by employing isoparametric elements and point matching. Differing from the traditional solutions using pulse basis, the one presented here was shown to be more accurate and stable, particularly in the case of scatterers having large refractive indices.

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Characteristic Impedance of a Tubular Dielectric Cylinder Covered with Conducting Arc Strips

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Abstract —**The characteristic impedance of a circular cylindrical dielectric tubular transmission line that is partially covered by thin conducting arc strips on the outer periphery is determined by conformal transformation. The variation of the characteristic impedance with the physical parameters is studied and some numerical results are presented.**

I. Introduction

Circular cylindrical dielectric waveguides that are partially covered with infinitesimally thin conducting coatings on the outer periphery have many potential applications in the design of transitions, baluns, and impedance transformers. Cylindrical stripline and cylindrical microstrip line fall into this category. Assuming that only the TEM mode exists, Wang [1] solved Laplace's equation by a dual series method and presented extensive results on the characteristic impedance of such lines. Joshi and Das [2] analyzed the problem of a cylindrical stripline with a homogeneous dielectric medium by a conformal transformation technique. Later, Joshi *et al.* [3] used the logarithmic transformation and reduced the problem of cylindrical stripline to an equivalent planar geometry. Recently, Zeng and Wang [4] also used conformal transformation to find expressions for the characteristic impedance in a closed form for cylindrical and elliptical striplines and microstrip lines with zero and finite-thickness strip conductors. Reddy and Deshpande [5] obtained a closed-form expression for the characteristic impedance of a cylindrical stripline with multilayer dielectrics.

In the present article, the transmission line consists of a hollow dielectric tube with two infinitely long thin conducting arc strips on the outer periphery, as shown in Fig. 1(a). Alternatively, the line can also take the complementary form shown in Fig. 1(b). The characteristic impedance is determined by first transforming the geometry to an equivalent planar geometry. In the planar form, the structure shown in Fig. 1(a) is similar to the coplanar stripline (CPS), while the geometry of Fig. 1(b) is similar to the coplanar waveguide (CPW) originally proposed by Wen [6]. The characteristic impedance of these lines can easily be found from the design equations given by Gupta *et al.* [7].

II. Theory

A hollow dielectric tube of permittivity ϵ_r, internal radius a, and external radius b is considered whose transverse cross section is shown in Fig. 1(a). Two infinitesimally thin conducting arc strips of width $b\phi$, symmetrically located with respect to the y axis, are located on the outer boundary of the dielectric. The geometry shown in Fig. 1(b) is complementary to that of Fig. 1(a), where the positions of slot and the arc strips are interchanged. The angular separation between the strips (or slots) is denoted by 2ψ. The two geometries of Fig. 1 can be transformed into planar geometries with the transformation function

$$w = \pi/2 + j\ln z \tag{1a}$$

where

$$z = x + jy \qquad w = u + jv \tag{1b}$$

refer to the variables in the original geometry and the transformed geometry, as shown in Fig. 2. The surfaces $\rho = a$ and $\rho = b$ transform to the straight lines v_1 and v_2, given by

$$v_1 = \ln a \tag{2}$$

$$v_2 = \ln b. \tag{3}$$

The distance between the two planes v_1 and v_2 becomes

$$h = \ln(b/a). \tag{4}$$

The width of the conducting strips (or slots) becomes

$$w_1 = \phi \tag{5}$$

and the spacing between them becomes

$$S = 2\psi. \tag{6}$$

The characteristic impedance of a coplanar waveguide (CPW) was calculated by Wen [6] by a quasi-static analysis and conformal mapping, where the dielectric substrate thickness was assumed to be sufficiently large to be considered infinite. For

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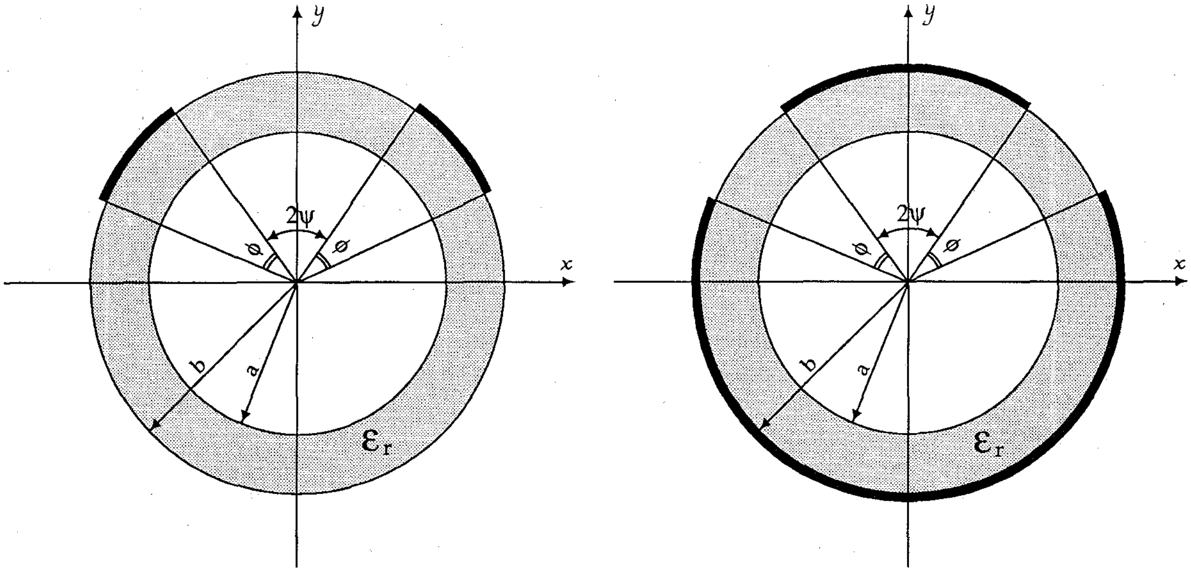

(a) (b)

Fig. 1. Cross sections of the transmission line.

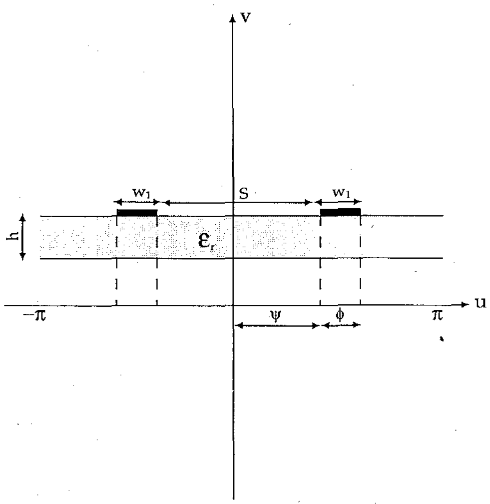

Fig. 2. Transformed planar geometry.

commonly used thicknesses, this assumption is valid for large values of the dielectric constant. A modification of the method studied by Wen is given by Davis *et al.* [8], and this model takes the finite thickness of the dielectric substrate into consideration. The quasi-static results for CPW due to Wen are modified by Gupta *et al.* [7] as

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_{re}}} \frac{K'(k)}{K(k)} \tag{7}$$

where

$$k = \frac{S}{S + 2w_1} \tag{8}$$

and $K(k)$ is the complete elliptical integral of the first kind. The corresponding expression for the geometry given in Fig. 1(a) is

$$Z_0 = \frac{120\pi}{\sqrt{\epsilon_{re}}} \frac{K(k)}{K'(k)}. \tag{9}$$

The ratio $K(k)/K'(k)$ can be approximated by the relations [7]

$$\frac{K(k)}{K'(k)} = (1/\pi)\ln\left[2(1+\sqrt{k})/(1-\sqrt{k})\right], \qquad 0.707 \leqslant k \leqslant 1 \tag{10a}$$

$$\frac{K(k)}{K'(k)} = \pi/\ln\left[2(1+\sqrt{k'})/(1-\sqrt{k'})\right], \qquad 0 \leqslant k \leqslant 0.707 \tag{10b}$$

where $k' = \sqrt{(1-k^2)}$.

The dielectric thickness is finite and the effective dielectric constant is given by [7]

$$\epsilon_{re} = \left[(\epsilon_r + 1)/2\right]\left[\tanh\left[1.785\log(h/w_1) + 1.75\right] + (kw_1/h) \cdot \left\{0.04 - 0.7k + 0.01(1 - 0.1\epsilon_r)(0.25 + k)\right\}\right]. \tag{11}$$

The results obtained by using (11) are considered very accurate for $h/w_1 \geqslant 1$ or, equivalently, $b/a \geqslant e^{\phi}$.

III. Results

The characteristic impedance Z_0 is calculated for different values of ϕ, b/a, ψ, and ϵ_r from (7) or (9), and some of the results are shown in Figs. 3 and 4. Fig. 3 shows the variation of the characteristic impedance for fixed values of the dielectric constant ($\epsilon_r = 17$) and the arc strip width ($\phi = 6°$) for varying values of the ratio b/a and the angle ψ for the two geometries shown in Fig. 1. The results for the geometry of Fig. 1(a) are shown by the full line and go with the scale on the left side of the graph, while corresponding results for the geometry of Fig. 1(b) are shown by dotted lines and the scale on the right side of the graph. Similarly, Fig. 4 shows the variation of the characteristic impedance for different values of strip width ϕ and angle ψ for fixed values of the permittivity ϵ_r and the ratio b/a. It may be noticed that the characteristic impedances of the two geometries approach one another for particular values of ϕ and ψ. In general, the characteristic impedance is higher for smaller arc strip widths and as the latter increases, the impedance decreases. It may be reiterated here that the analysis presented in this article is a quasi-static one where only the TEM mode is assumed to be present. This by itself is not a limitation, since the dispersion analysis carried out earlier for these structures indicated that the

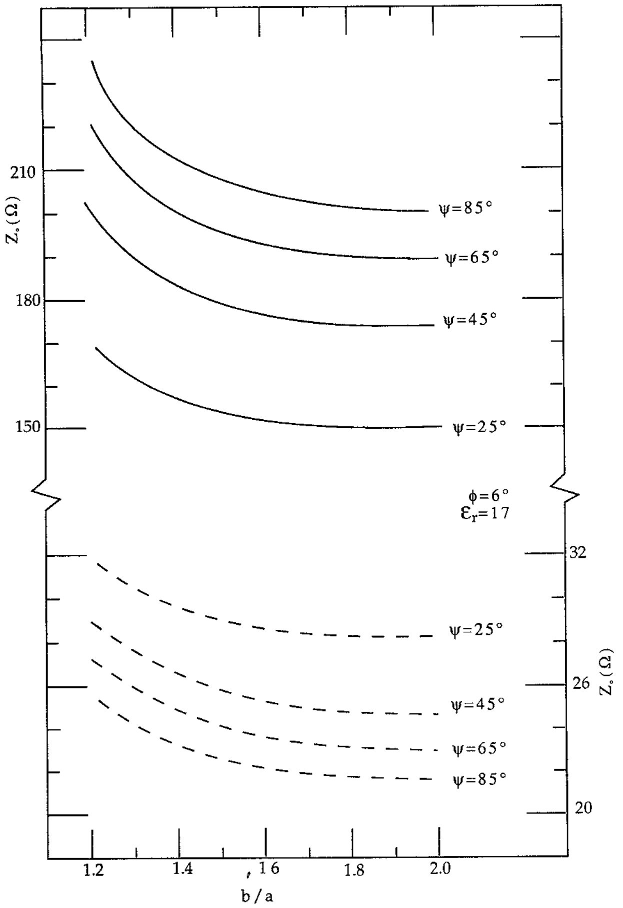

Fig. 3. Variation of the characteristic impedance with b/a and ψ.

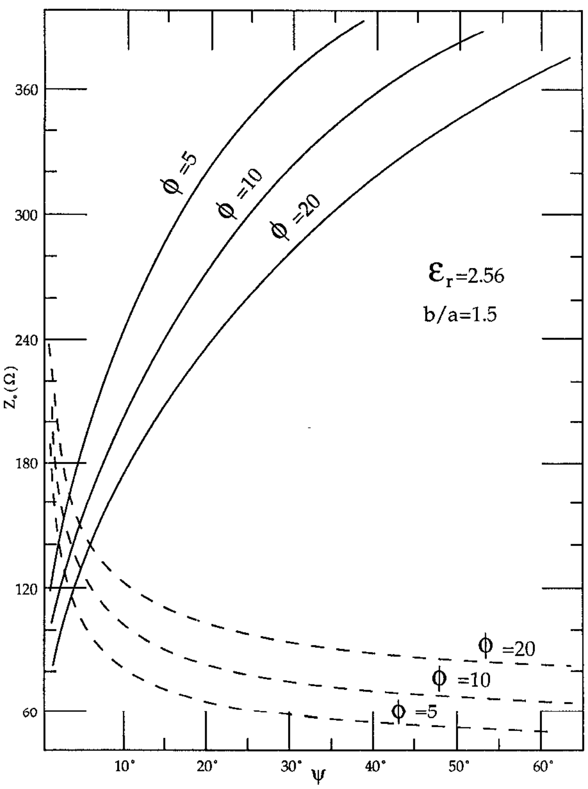

Fig. 4. Variation of the characteristic impedance with ψ and ϕ.

quasi-TEM analysis may be used for frequencies below X-band frequencies [7]. The solution is based on conformal transformation, and the validity of the method depends on the relative magnitudes of the ratio b/a and the arc strip width ϕ. These preliminary results suggest that the geometry shown in Fig. 1(b) is useful for low-impedance applications whereas the geometry of Fig. 1(a) provides a high-Z_0 transmission line. The feeding arrangement is very similar to that of a slotline, usually from a coaxial line.

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Spectral-Domain Immittance Approach for the Propagation Constants of Unilateral Finlines with Magnetized Ferrite Substrate

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***Abstract* —In this paper, the admittances of TM and TE modes in ferrite substrate are given, and the propagation constants for the forward and backward directions of the dominant mode in unilateral finlines with magnetized ferrite substrate are obtained. The solution is obtained by applying the transverse equivalent transmission line in the spectral domain and Galerkin's method. Numerical results are presented which can be used in designing a finline displacement isolator.**

I. Introduction

Recently, finlines have become attractive for millimeter-wave integrated circuit applications. Different types of finline configurations, such as unilateral, bilateral, and antipodal on isotropic substrates, have been extensively studied and employed in practice. Realization of nonreciprocal devices in finline techniques is also of interest in the millimeter-wave range because of the relative compactness and integrability of the devices compared to conventional ferrite-loaded waveguide. Beyer *et al.* have reported the experimental investigation of finline isolators and circulators [1], [2]. The theoretical treatment in [2] is also useful; however, it

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